Time-varying MTR Codes for High Density Magnetic Recording

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ABSTRACT— Maximal transition run (MTR) codes eliminate the dominant error-event in high density magnetic recording by allowing at most two consecutive magnetic transitions. The maximum rate of such a code is upper bounded by the Shannon capacity 0.879. Unfortunately, the coding gain for MTR codes has not proven to be very significant since the distance benefit due to removing the dominant error-event barely makes up for the rate loss. In this paper, we describe a class of higher rate codes that eliminate the dominant high density error-event and provide improved coding gain. The new less restrictive constraints allow at most three consecutive transitions and have Shannon capacities as high as 0.925. Two high rate block codes with time-varying MTR constraints are presented. One code is rate 8/9 with block 9 and run-length constraint (d=0, k=11) and the other is rate 9/10 with block length 10 and (d=0, k=16).

I. INTRODUCTION

One of the biggest challenges in magnetic recording for disk drive systems is the doubling of capacity every 18 months. With increased recording density comes greater intersymbol interference that requires more complex detection schemes to achieve high performance. A Viterbi detector is used to estimate the recorded sequence by selecting the one associated with the noiseless channel output sequence closest to the noisy channel output. Therefore, detector performance is dominated by the minimum distance between any two noiseless channel output sequences. Codes that eliminate the dominant error-events increase the minimum distance between the channel output sequences produced by any two allowed sequences. However, just constraining the recorded sequences is not enough to achieve the distance gain. The detector must be designed to select only those sequences allowed by the constraint [1]. The coding gain provided by the code is the difference between the distance gain and the rate loss required to overcome the degraded performance at the higher recording density associated with the lower code rate.

Maximal transition run (MTR) codes have been suggested as a means of achieving coding gain for high density magnetic recording [2,3]. An MTR code eliminates the minimum distance error-event at high densities by restricting the recorded sequence to have at most two consecutive transitions. This paper presents a new class of codes that have the same distance enhancement as a standard MTR code, but achieve higher coding gain due to higher code rates.

In saturation magnetic recording, a two-level write current signal is recorded onto the magnetic medium. The nomenclature used to denote the recorded symbols is referred to as Non-Return to Zero (NRZ). A +1 corresponds to magnetization in one direction and a −1 corresponds to magnetization in the opposite direction. A magnetic transition occurs between two symbols of opposite polarity. No transition occurs between adjacent symbols of the same polarity.

Codes that place restrictions on the recorded sequence are used to improve the reliability of the detected signal. In magnetic recording, the most commonly used codes restrict the minimum and maximum spacing between transitions. These codes are referred to as (d, k) run-length limited (RLL) modulation codes. The d and k constraints indicate the minimum and maximum number of consecutive non-transitions, respectively. The k constraint is used to improve the ability of adaptive loops to track variations in the timing and the gain by insuring that transitions occur frequently. For channels with spectral nulls, an additional constraint is usually required to limit the path memory length in the Viterbi detector by limiting the length of the minimum distance error-events. Codes with this additional constraint are referred to as (d, G/I) RLL codes, where G represents the k constraint on the global sequence and I represents the k constraint on the even and odd interleaved sequences [4].

At high densities, the dominant error-event corresponds to mistaking the write current sequences 1−1 1 and −1 1−1 for each other [3]. This error-event is denoted by 2−2 2. Codes with d greater than zero eliminate this error-event as well as other likely error-events providing a significant increase in the minimum distance. However, the distance gain is offset by a significant rate loss penalty, since the capacity of the d=1 constraint is 0.694 and the capacity of the d=2 constraint is 0.551.

An effective method of generating high rate distance enhancing codes, referred to as forbidden list codes, was introduced by Karabed and Siegel [5]. These codes are designed so that if two sequences are separated by a dominant error-event then a one or both of the sequences is forbidden in code. In [5], forbidden list codes with rates 0.8 and above are shown to achieve the same distance gain as a rate 2/3 d=1 code on the E^2PR4 channel.

Here, we use the same technique to show that codes with capacities as high as 0.925 eliminate the same dominant high density error-event as an MTR code with capacity 0.879. In addition, we show that time-varying forbidden lists can be used to construct a rate 8/9 (d=0, k=11) block code with block length 9 and a rate 9/10 (d=0, k=16) block code with block length 10.

II. MTR CONSTRAINT

An error-event occurs when the detected sequence is not equal to the recorded sequence. In order to have an isolated

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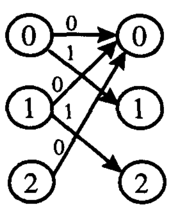
error-event, the recorded and detected sequences must have the same symbols before and after the error-event. Therefore, an isolated 2–2 2 error-event has an error sequence with an infinite number of zeros preceding and following the error-event, i.e., ...0 0 2–2 2 0 0 ... .

The 2 –2 2 error-event involves mistaking the two write current sequences 1 –1 1 and –1 1 –1. Since the write current symbols directly preceding and following the error-event must be the same for the recorded sequence and the detected sequence, there are four possible cases;

CASE 1:	1 1 –1 1 1	2 consecutive transitions
	1 –1 1 –1 1	4 consecutive transitions
CASE 2:	–1 1 –1 1 –1	4 consecutive transitions
	–1 –1 1 –1 –1	2 consecutive transitions
CASE 3:	–1 1 –1 1 1	3 consecutive transitions
	–1 –1 1 –1 1	3 consecutive transitions
CASE 4:	1 1 –1 1 –1	3 consecutive transitions
	1 –1 1 –1 –1	3 consecutive transitions.

In each of the above cases, the error-event occurs when one of the two sequences is the recorded sequence and the other is the detected sequence. Since at least one sequence in each of the four cases contains three or more consecutive transitions, the MTR constraint eliminates the dominant error-event by eliminating at least one of the two sequences in each case. Therefore, the MTR code satisfies the condition that no two coded sequences are separated by the dominant high density error-event.

The MTR constraint is most easily described in terms of the Non-Return to Zero Inverse (NRZI) notation, where a 0 corresponds to no transition and a 1 corresponds to a transition. The encoder produces sequences in NRZI notation that are converted to NRZ sequences by a precoder with transfer function 1/(1+D) (mod 2). The MTR constraint does not allow the recorded sequences to contain three consecutive transitions, i.e., the NRZ sequences 1–1 1–1 and –1 1–1 1 are forbidden. Therefore, the encoded sequence at the input to the precoder cannot contain the NRZI sequence 1 1 1. The MTR trellis and state transition matrix are shown in Fig.1, where the branches are labeled with NRZI encoded symbols.



$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Fig. 1: Trellis and transition matrix for MTR.

The Shannon capacity of a constraint graph is the log base 2 of the largest real eigenvalue of the state transition matrix [4]. The capacity is an upper bound on the rate for a code satisfying that constraint. The capacity of the MTR constraint is 0.879.

For channels with 8 or more states, the MTR constraint removes branches and/or states from the Viterbi detector for the channel providing coding gain without adding complexity. In an 8-state Viterbi detector, states 1–1 1 and –1 1–1 each have only one branch entering and one branch leaving. In a 16-state Viterbi detector, states 1–1 1–1 and –1 1–1 1 are simply removed, leaving only 14 states in the detector trellis.

III. TIME-VARYING MTR CONSTRAINT

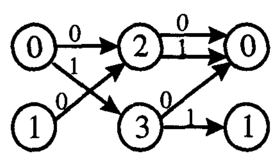
In this section, we show that the dominant high density error-event can be eliminated with a higher capacity time-varying MTR constraint that allows at most three consecutive transitions. Referring back to the four possible cases in Section II, forbidding four consecutive transitions eliminates one of the two sequences in both Cases 1 and 2. In Cases 3 and 4, the two sequences each have three consecutive transitions that end at consecutive time periods, as follows:

CASE 3:	–1 1 –1 $\underline{1}$ 1	3 transitions end at time j
	–1 –1 1 –1 $\underline{1}$	3 transitions end at time j+1
CASE 4:	1 1 –1 1 –$\underline{1}$	3 transitions end at time j+1
	1 –1 1 –$\underline{1}$ –1	3 transitions end at time j.

Any code that forbids four consecutive transitions and does not allow three consecutive transitions to end at consecutive time periods eliminates the dominant high density error-event. Condition 1 restates this constraint in terms of NRZI notation.

CONDITION 1: The sequence 1111 is forbidden all of the time and the sequence 01110 is forbidden from appearing in any two NRZI encoded sequences offset by one symbol.

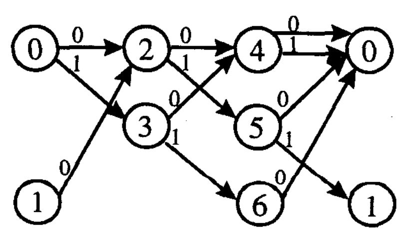
One way of satisfying Condition 1 allows three consecutive transitions to end at every other time period. This constraint is referred to as a modulo 2 time-varying MTR (TMTR) constraint. The time-varying trellis and the state transition matrix for this constraint are shown in Fig. 2. The Shannon capacity is 0.916.



$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Fig. 2: Trellis and transition matrix for mod 2 TMTR.

Another way to satisfy Condition 1 allows three consecutive transitions to end at every third time period. This constraint is referred to as a modulo 3 TMTR constraint. The time-varying trellis and the state transition matrix for this constraint are shown in Fig. 3. The capacity is 0.903.



$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig. 3: Trellis and transition matrix for mod 3 TMTR.

Since these constraints have capacity greater than 0.889, rate 8/9 TMTR codes with the same distance enhancement as standard MTR codes are possible. A rate 8/9 block code with block 9 is desirable for limiting error propagation in an outer error-correction code with 8-bit symbols. A rate 8/9 TMTR block code with block length 9 consists of 256 codewords of length 9 that satisfy Condition 1 when concatenated.

The mod 3 TMTR constraint has only 255 words and not the required 256 words. Although the mod 2 TMTR constraint has more than 256 words, it does not have 256 words of length 9 that can be freely concatenated, due to the even constraint period and the odd block length.

In order to create a rate 8/9 TMTR block code with block length 9 the properties of both the mod 2 and mod 3 constraint graphs were combined. The resulting TMTR constraint shown in Fig. 4 has period 9, which matches the block length of the code. The trellis contains 267 words and nine of these words violate a (d=0, k=11) constraint. The TMTR constraint is satisfied because four consecutive 1's are not allowed and three consecutive 1's can only end at states 2, 7, 11, and 15, which are all separated in time by at least one time period, even when concatenated. Assuming that there is a modulo 9 counter synchronized to the data, the three transitions in a row can end at times 0, 3, 5, and 7 relative to the counter. The trellis allows codewords to end with at most two consecutive 1's and to begin with at most one 1.

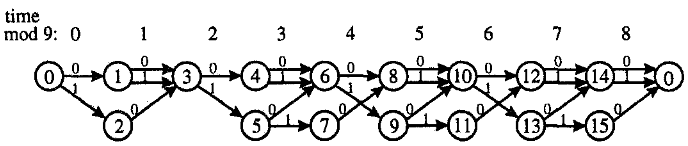


Fig. 4: The trellis for a rate 8/9 TMTR block code.

The rate 8/9 TMTR block code requires a time-varying Viterbi detector. Like a standard MTR code, the TMTR code only requires removing branches from the detector trellis for the channel. In a 16-state Viterbi detector, the states 1−1 1−1 and −1 1−1 1 are removed from the detector trellis for those time periods that do not allow three transitions. Removing these states in a time-varying manner can be implemented in the selection circuit within the add-compare-select units for four states. The selection circuit for state 1−1 1−1 is modified to always select the path from state 1 1−1 1. Similarly, the selection circuit for state −1 1−1 1 is modified to always select the path from state −1−1 1−1. The selection circuit for state 1−1 1 1 is modified to select the path from state 1 1−1 1, if three consecutive transitions are not allowed to end during the previous time period, and to select the path with the minimum metric, otherwise. Similarly, the selection circuit for state −1 1−1−1 is modified to select the path from state −1−1 1−1, if three consecutive transitions are not allowed to end during the previous time period, and to select the path with the minimum metric, otherwise.

Many researchers have recently published similar work on high rate distance enhancing codes. Bliss [6] independently discovered the mod 2 and mod 3 time-varying MTR constraints and constructed a rate 8/9 mod 2 TMTR block code with block length 18 by alternating between two different rate 8/9 block codes with block length 9. Since the two subordinate block codes are decoded independently, this block code with block length 18 has the same minimal error propagation that is exhibited by a block code with block length 9. However, the encoder and decoder logic is potentially doubled due to the two alternating codes. Concurrently, the rate 8/9 TMTR block code constraint illustrated in Fig. 4 was independently discovered by Moision, Siegel, and Soljanin [7].

IV. HIGHER RATE CODES

Constraints with higher capacity are obtained by considering more symbols preceding and following the dominant error-event. Next, we extend the four cases in Section II by one symbol on each end to obtain 16 cases. Of the 16 cases, only 2 cases do not contain at least one sequence with four consecutive transitions. They are

CASE 5: $-1\ -1\ \ 1\ -1\ \ \underline{1}\ \ 1\ \ 1$
$\qquad\quad -1\ -1\ -1\ \ 1\ -1\ \ \underline{1}\ \ 1$
CASE 6: $\ 1\ \ 1\ \ 1\ -1\ \ 1\ -\underline{1}\ -1$
$\qquad\quad \ 1\ \ 1\ -1\ \ 1\ -\underline{1}\ -1\ -1.$

Any code that satisfies Condition 2 which is stated in terms of NRZI notation eliminates the dominant high density error-event.

CONDITION 2: The sequence 1111 is forbidden all of the time and the sequences 011100 and 001110 are forbidden from appearing time aligned in any two NRZI encoded sequences.

Note that Condition 2 is less restrictive than Condition 1, since for example the sequence 1011101 is allowed at any time.

A forbidden list constraint that forbids 001110 and 1111 all of the time satisfies Condition 2. This constraint requires that three consecutive transitions are preceded by exactly one non-transition and has capacity 0.913. The 2−2 2 error-event cannot appear by itself, since the NRZI sequences 01110 and 10111 (i.e. NRZ sequences −1−11−111 and 1−1−11−11, respectively) differ by the NRZ error sequence −2 0 2 −2 2 0.

This principle can easily be extended to show that the dominant error-event is eliminated by forbidding 100(00)1110 and 1111 all of the time, where the notation (00) means that 00 can be repeated any number of times such that the code forbids an even number of non-transitions preceding three consecutive transitions. Since this constraint has capacity 0.925, codes with rates as high as 12/13 are possible.

Next, we show that a rate 9/10 block code can be constructed using a time-varying MTR constraint that satisfies Condition 2, where the TMTR constraint eliminates the encoded NRZI sequences 0011100, 1011100, and 0011101 every other time period and eliminates 1111 all of the time. The capacity of this TMTR constraint is 0.923. Fig. 5 shows a time-varying trellis for a block code with block length 10 that allows the sequences 001100, 1011100, and 0011101 to end only at times 0, 3, 5, and 7 relative to a modulo 10 counter synchronized with the data. From the constraint graph containing 525 words, a rate

9/10 block code with (d=0, k=16) is constructed by removing the seven words that violate the run-length constraint.

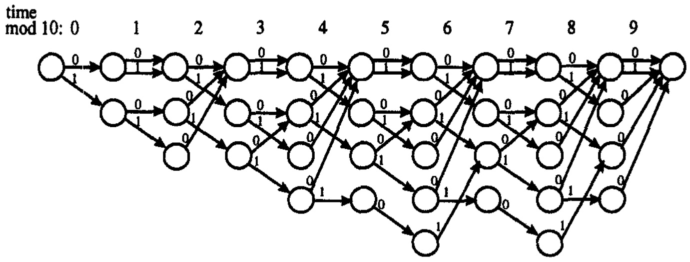


Fig. 5: The trellis for a rate 9/10 TMTR block code.

V. SIMULATION RESULTS

A computer simulation was used to compare the performance of a rate 8/9 TMTR code to that of a rate 16/17 (d=0,G/I) RLL code with no distance enhancement. The simulator used linear superposition of a measured step response from an MR head and additive white Gaussian noise to approximate the playback signal of a magnetic recording system. Using a 25 tap finite impulse response (FIR) filter, the noisy playback signal was equalized to either EPR4 or to a minimum mean-square error decision feedback equalizer (MMSE-DFE) target with seven nonzero samples [8]. When equalized to an EPR4 target, the simulator used an 8 state Viterbi detector. When equalized to a MMSE-DFE target, the simulator used a reduced-state sequence estimation (RSE) detector with 16 states and 2 feedback taps, which is similar in performance to a 64 state Viterbi detector matched to the MMSE-DFE target [8].

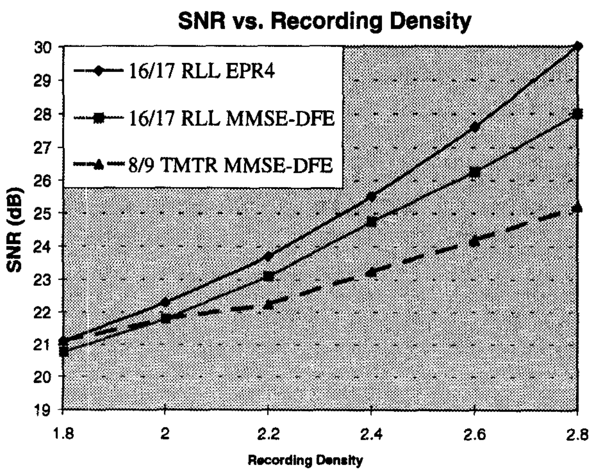


Fig. 6: The SNR in dB required to achieve a 10^{-5} bit error rate versus recording density.

Figure 6 shows the signal-to-noise ratio (SNR) necessary to achieve a bit error rate of 10^{-5} as a function of recording density for a rate 8/9 TMTR code and for a rate 16/17 RLL code applied to a MMSE-DFE channel. Note that the MMSE-DFE target is not a fixed target; it is optimized for each recording density. The performance of a rate 16/17 RLL code applied to an EPR4 channel is included for reference.

In the figure, recording density is defined as the pulse width at 50% of peak amplitude divided by the duration of a single information bit. The figure shows the TMTR code providing little benefit at recording densities below 2.2 and increasing benefit at higher densities. At 2.8 the TMTR code on the MMSE-DFE channel provides nearly 3 dB of performance benefit over the rate 16/17 RLL code on the MMSE-DFE channel and nearly 5 dB improvement over the rate 16/17 RLL code on the EPR4 channel.

VI. CONCLUSION

In this paper, we show that the dominant error-event in high density magnetic recording can be eliminated by much looser constraints than the MTR constraint which eliminates all instances of three consecutive transitions. As a result, high rate codes that produce significantly higher coding gain than standard MTR codes are possible. Time-varying MTR block codes with rates 8/9 and 9/10 were constructed.

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